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A STAR FIELD MAPPING SYSTEM FOR DETERMINING THE

ATTITUDE OF A SPINNING PROBE

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NASA Langley Research Center Langley Station, Hampton, Va.

Presented at the Aerospace Electro-Technology Symposium of the International Conference and Exhibit on Aerospace Electro-Technology

GPO PRICE \$ \_\_\_\_\_

Hard copy (HC)

ff 653 July 65

Phoenix, Arizona April 19-25/1964



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### A STAR FIELD MAPPING SYSTEM FOR DETERMINING THE ATTITUDE OF A SPINNING PROBE

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ABSTRACT

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In certain space experiments precise knowledge of the orientation of the attitude angles of a spin stabilized probe is required. This requirement has resulted in the development of a unique technique of star mapping herein discussed.

The star mapping is accomplished by optically scanning a band of the star field about the vehicle's equator. Determination of the orientation of the vehicle's spin axis and roll angle to the celestial sphere is accomplished by cross correlation of the scanned star map with a known reference map of the celestial sphere.

The spinning motion of the vehicle causes star images to pass over a combined optical-reticlesensor which is configured to generate two (2) groups of coded pulses at the output of the sensor. The amplitudes of signal out of the optical sensor will be proportional to the spectral radiance of the scanned stars since the action of the reticle is to modulate the radiation received by the optical sensor and thus classification of stars according to their visual magnitude is possible. The time of the occurrence of the two (2) pulse groups is related to the azimuth angle to the star and the time separation of the pulse groups is related to the star's elevation angle.

This paper includes a description of the system configuration and system environment. System performance is discussed from the viewpoint of parameters which limit performance characteristics of the system. A typical design is considered and estimated performance characteristics are presented.

SUMMARY

A star mapping system is described which passively maps a band of the star field about the equator of a spinning spacecraft and provides a simple means for determining the spacecraft attitude. Factors which affect the performance of the star mapping system are discussed and a performance analysis is presented for a system designed to map stars of third magnitude and brighter with an angular resolution of 0.025°.

### INTRODUCTION

Certain space experiments which utilize spin stabilized spacecraft require precise knowledge of the spacecraft attitude angles. A promising approach to this problem is to determine the attitude of the vehicle with respect to the celestial sphere. This paper describes a unique technique for this type of attitude determination in which a strip map of the star field surrounding the vehicle's equator is generated by a passive star telescope which scans the celestial sphere as the vehicle spins. Attitude angles can then be determined by correlating the generated star map with known stellar positions. Using this type of star

field correlation, attitude angle measurements are possible with as few as two identifiable stars for a body spinning about a principal axis.

### PRINCIPLE OF OPERATION

The vehicle under consideration is a spin stabilized probe. The basic star scanner consists of a telescope, a recticle, and a photomultiplier tube. The telescope is mounted in the probe with the optical axis normal to the spin axis of the vehicle. The reticle is centered in the focal plane of the telescope with the photomultiplier mounted behind the reticle. The reticle is opaque with two groups of transparent slits. The slits of one group are parallel to the vehicle spin axis and the second group is placed at a known angle to this axis. This configuration is shown in figure 1. As indicated by this figure the spinning motion of the vehicle causes the star image to pass over the reticle and consequently the input to the photomultiplier will be a series of coded pulses of radiant energy for each slit group. The number of pulses is dependent upon the number of transparent openings of the reticle. The code possesses special autocorrelation features by which synchronization may be obtained. This pulse coding principle can provide unambiguous indications of a star crossing. The time of occurrence of the two pulse groups provides a measure of the azimuth angle to the star and the time separation of a single pair of pulse groups provides a measure of the elevation angle to the star. The amplitude of the pulses of a particular pulse group will provide a measure of the intensity of the scanned star. The output of the photomultiplier tube is telemetered to a ground station for data processing by which the vehicle attitude is determined.

### System Description

The input signal to the system and system characteristics including the ground data handling scheme are shown in figure 2.

The star signal which is scanned by the telescope is dependent upon the energy density and spectral characteristics of the star. Telescope parameters which affect this signal are field of view, which determines whether or not the star is viewed; aperture of the telescope; spectral characteristics, which limit the spectrum of acceptable energy; and efficiency of the optics, which determines the efficiency of transfer of energy to the focal plane of the telescope. The combination of the reticle and vehicle spin causes the signal at the detector to appear as a sampled signal. The signal as sensed by the photomultiplier is a function of the spectral response characteristics and gain of the photomultiplier tube. The airborne electronics operates on the signal with a gain compatible with transmitter requirements and a band pass compatible with the sampling frequency produced by the reticle and vehicle spin combination.



The signal detection scheme in the ground data handling equipment is one in which the signal amplitude is compared to a variable threshold value (preselected to agree with an amplitude resulting from a star of a given brightness) for signal detection and classification purposes. This classification will also include time of occurrence of the star scanned in the field of view and time elapsed between repeated scans of the same star in a complete revolution of the vehicle. Measurement of the elevation angle to the scanned star may be determined by noting the elapsed time between sucsessive pulse groups produced by a single star. These times may be converted to angular measurements through a knowledge of the vehicle spin rate and the geometry of the reticle. This time-angle relationship is illustrated in figure 3. In this figure the time of occurrence of a pair of pulse groups is indicated by  $t_1, t_2, \ldots$ , the time lapse between a pair of pulse groups is indicated by  $\Delta t_1$ ,  $\Delta t_2$ , . . ., and vehicle spin rate is noted as w.

### Data Processing

The time of occurrence of a pulse group generated by a scanned star is determined through the process shown in figure 4. The signals from the receiver pass through the threshold circuit which quantizes the signal into two levels. This binary signal is then compared to a reference code (a replica of the reticle code) in the decoder. When the proper code sequence is recognized, the time of occurrence of this recognition is recorded in digital form and the pulse amplitude is also recorded by activating the A/D convertor at the time of pulse group recognition. Adjustment of the threshold provides a convenient way of restricting the response of the system to a few of the brighter stars in the scanned field. Figure 3 illustrates the computation necessary to convert the digital record to a star map with an azimuth dimension of 3600 and which is bounded in elevation by the vertical field of view of the telescope. The result of this data processing is the generation of a map of the scanned star field with classification of the brightness of the stars and a measure of their azimuth and elevation angles in the scanned field of view.

### Data Interpretation

The generated star map represents a twodimensional description of the orientation of the field of view of the star mapper in the celestial sphere. The orientation of this field of view and the direction of the vehicle spin axis may be determined by a two-dimensional cross correlation of the generated star map with a reference star map. To minimize processing time the reference map may be constructed to cover that portion of the celestial sphere which is expected through a priori knowledge. This reference map must be large enough to accommodate the field of view of the telescope and any coning or nutation about the vehicle spin axis due to unbalance of moments of inertia of the vehicle about the pitch and yaw axes. Nutation can cause considerable difficulty in the interpretation of the data and a greater number of star sightings may be required for complete attitude determination if nutation amplitudes are appreciable.

# SENSITIVITY AND NOISE CONSIDERATION

The performance of the described system is best determined by the ability of the system to identify an adequate number of stars in the scanned star field for star field identification and accuracy in measuring the orientation of the star field with respect to the field of view of the vehicle. The orientation accuracy is a function of angular resolution of the optics whereas the identification problem is a function of signal detectability.

### Desired Characteristics

For positive identification of a scanned star field it is necessary to insure that an adequate number of detectable stars will be scanned in a given star field. Figure 5 represents the approximate number of stars brighter than a given magnitude, where the number of stars are referenced to a field of view of one square degree (ref. 1). Consideration of the requirement of at least two identifiable stars and the knowledge that our star field is 360° in the azimuth direction can lead to a reasonable selection of the optical field of view. For example, if a 6° vertical field of view is selected, the number of stars brighter than a given magnitude in the resultant field of view of 2,160 square degrees are as shown in figure 6. From this figure it can be seen that the brightness of stars under consideration is on the order of third magnitude and brighter. Ideally the system would be designed to detect or accept all scanned stars brighter than the selected magnitude and sharply reject those stars which are dimmer than this magnitude. This approach would insure detection of an adequate number of stars for star field identification and yet reduce the ambiguities which would arise for detection thresholds set for the dimmer stars in the field.

### Telescope and Detector Characteristics

The star signal as seen by the photomultiplier is dependent upon the energy density spectrum of the star signal and the spectral characteristics of the optics and photomultiplier. A rough approximation to the stellar spectra can be made by estimating a source temperature and assuming a black-body spectrum. Since we are interested in the brighter or more prominent stars of a given star field we need to examine the spectral characteristics of stars according to their visual classification. Representative equivalent source temperatures for stars of third magnitude to zero magnitude are of the order of  $6,000^{\circ}$  K to  $10,000^{\circ}$  K. It can be shown that for these temperatures the spectral characteristics of the optics and photomultiplier should have a peak between approximately 3,000 to 5,000 angstroms (ref. 1). Selection of the telescope aperture and photomultiplier tube gain are not discussed here but are considered to be of nominal values required for suitable signal amplification.

### Signal and Noise Characteristics

The principal problem involved in a system of this type is the problem of signal detectability in the presence of the noise of the system. Before proceeding to the problem of signal detection it is of interest to examine the characteristics of signal and noise as seen by the photomultiplier.

Noise sources .- The noise sources considered in this paper are: external noise due mainly to scattered and direct starlight, and galactic light; noise of the photomultiplier tube; and electronic noise of the system. The noise due to starlight is considered to originate from the dimmer stars, that is, the nonprominent stars. Galactic light is starlight emitted or scattered by interstellar dust in the Milky Way. This external light is considered to be of a diffuse nature and hence the noise as seen by the photomultiplier is dependent upon the effective field of view of the telescope. Effective field of view is defined here as the total clear opening of the reticle in the focal plane. The photomultiplier tube is a quantum detector being sensitive to the rate of interception of light quanta arriving at the photocathode rather than rate of energy arrival or power. For most photomultipliers the quantum efficiency varies continuously over the spectral response region of the tube. Since the probability of emission of an electron, or quantum efficiency, is a function of frequency of the exciting photon, any enhancement of photon noise by the photocathode should be calculated by an integral for other than monochromatic light.

Since the release of electrons from the photocathode is related to the quantum efficiency or to a statistical process it is important to establish the type of distribution associated with this photocathode current. The Poisson distribution agrees with experimental results and is generally assumed. Now by definition of the system inputs, the optical parameters, and the photomultiplier tube, signal detectability may be examined in view of the number of electrons due to signal and noise at the photocathode. The noise considered here will be due to background noise, photomultiplier dark current, and signal induced noise.

In general, noise sources such as the electronic noise of the airborne equipment and RF noise are negligible as compared to the other noise sources discussed above.

Background noise .- Stellar background pertains to the quantity and quality of the stellar radiations. Quantity refers to the number of stars and their irradiance. The quality of radiation refers to the spectral distribution of the stellar irradiance. Since the total number of dim stars is immeasurable, the irradiance description must be discussed in terms of the distributed radiance of the celestial sphere. The purpose of this discussion is to describe the background noise that would affect detection of a given star. The background light as viewed by the telescope may be considered as an equivalent signal in terms of total integrated starlight in effective number of tenth magnitude stars per square degree of effective field of view (ref. 2). This equivalent signal may be handled in a manner similar to the star signal to determine the effective noise level as seen by the photomultiplier.

Signal and noise parameters. In the determination of the number of photocathode electrons (due to signal and noise) which are produced in a given interval of time, there are certain system parameters which must be defined. The following three equations express the average number of electrons per unit time for signal  $\overline{N}_B$ , and dark current  $\overline{N}_T$ , respectively:

$$\overline{S} = K_1 K_2 AG \frac{\alpha}{m} f(m)$$
 (1)

$$\overline{N}_{B} = K_{1}K_{2}K_{3}AG \frac{\alpha^{2}}{\omega} \in n$$
 (2)

$$\overline{N}_{\tau} = \frac{1}{2} (K_2 \eta G)^2 \frac{\alpha}{\omega}$$
 (3)

where

 $K_1$  stellar constant of 2.1 × 10<sup>-10</sup> lumens/cm<sup>2</sup> for a star of zero magnitude

 $K_2$  1/1.6 × 10<sup>-19</sup> electrons per ampere second

K<sub>3</sub> background light in equivalent number of tenth magnitude stars per square degree of the effective field of view of the telescope

A effective telescope aperture in cm<sup>2</sup>

G photomultiplier sensitivity in μ amp/lumen

width of a reticle transparent slit in degrees (also the desired angular resolution)

w spin rate of the vehicle in degrees/second

€ vertical field of view

 $\rm f(m)\approx 2.5^{-m}$  (this term and  $\rm\,K_{1}\,$  determine the number of lumens for a star of magnitude m)

 $\eta$  equivalent tube noise in lumens  $\sec^{1/2}$ 

 $\frac{\alpha}{m}$  unit sample time or observation time

n equivalent number of reticle transparent openings of width  $\alpha$ 

Total expected signal in a unit sample time is defined as

$$\overline{S}_{m} = \overline{S} + \overline{N}_{B} + \overline{N}_{T} \tag{4}$$

### Coding and Signal Detection

This discussion considers the enhancement of detection through the use of multiple transparent slits in the reticle where each slit will have a width defined by the angular resolution requirement of the system. By proper design of the slit arrangement this technique may be extended to provide noise suppression. This scheme involves arrangement of the slits in a psuedo random pattern. The output of the photomultiplier will now be a pulse coded modulation of the star signal. The nature of this signal is a series of pulses as generated by the passage of the star image over the pair of reticles in the focal plane of the optical system. The amplitude of these pulses as discussed earlier is a function of the brightness of the star and the response of the optical system. The widths of these pulses are determined by the angular width of the transparent slits of the reticle and the spin rate of the vehicle. The principle used here

is to continuously correlate a reference code (identical to the code of the reticle) with the output of the photomultiplier. When the correlation process indicates sufficient agreement between the reference code and the coded signal there will be an indication of signal detection with a confidence level determined by the acceptable level of correlation. This feature can provide unambiguous determination of the times of occurrence of the coded star signals to the resolution afforded by the optical system.

In the development of the signal detection technique the model of the physical situation is defined as shown in figure 7. In this figure the sampler represents the PCM effect of the coded reticle and the spinning vehicle. The function of the amplitude selector and  $T_{1}$  (first threshold) is to select or gate signals which are greater than a preselected value. The function of the decoder or correlator is to continuously compare a reference code (a binary replica of the reticle code) to the coded signal, with r representing the number of agreements in a given code sequence. The function of the comparator and  $T_2$  (second threshold) is to act as a gate to permit acceptance of a preselected correlation or agreement count as an indication of a signal representing a star of a given magnitude or brighter.

If the characteristics of the signal and noise are known at the input to the amplitude selector and the first threshold is defined, the probability of accepting or rejecting the signal level of any given sample at the amplitude selector may be determined. Designating this probability as  $P\left(\overline{S}_T \geq T_1\right)$ , the probability of the decoder threshold being exceeded may now be examined.

Since the decoding process is a correlation technique this probability is defined as the probability that the correlation or agreement count exceeds a preselected level of agreement. This probability is designated here as  $P(r \geq T_2)$  where r is the number of agreements in a correlation and  $T_2$  is a preselected number of agreements in a code sequence. The parameter  $P(r \geq T_2)$  may be thought of as a probability of detection in the case where a signal is present and a probability of false alarm in the case where noise only is present.

In the detection technique under consideration a reference code of n elements is assumed and the code is defined as a two level binary function. The code is further defined to contain an equal number of elements of each state such as M ones and M zeros. The time duration of a single element is determined by the sampling interval. In the preparation of the coded signal for the decoding process the amplitude selector and the first threshold convert the signal to two levels, that is, each signal level in a sample interval exceeding T1 will be designated as a one, otherwise the signal level of that particular sample interval will be a zero. For a given signal level and noise condition and a given number of code elements, a simplified diagram of the photomultiplier signal positioned in register with an example reference code is as shown in figure 8(a).

Degree of correlation of a single sample pair. The distribution of signal values in a sample interval has previously been described as a Poisson distribution but for ease of analysis the distribution hereafter will be treated as a normal distribution, with the first and second order statistics being those of the Poisson distribution. Inaccuracies introduced by this assumption will be considered negligible so long as the expected number of electrons per unit sample is ten or greater. The amplitude intervals are assumed to be as shown in figure 8(b) and the error in levels of the samples which exists at the in-register position of the signal and code are assumed to be distributed according to a normal curve. With this assumption the probability that a noise-only sample will result in a zero is

$$P_{OO} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{Z_1} e^{-\frac{Z^2}{2}} dZ$$
 (5)

where

$$Z_{\perp} = \frac{T_{\perp} - \left(\overline{N}_{B} + \overline{N}_{\tau}\right)}{\sqrt{\overline{N}_{B} + \overline{N}_{\tau}}} \tag{6}$$

Similarly the probability that a sample containing signal plus noise will result in a value of one is

$$P_{11} = \frac{1}{\sqrt{2\pi}} \int_{Z_2}^{\infty} e^{-\frac{Z^2}{2}} dZ$$
 (7)

where

$$z_2 = \frac{\overline{x}_1 - \overline{s}_T}{\sqrt{\overline{s}_T}}$$
 (8)

Now  $P_{OO}$  and  $P_{l,l}$  are defined as the probability of agreement of a single sample pair of noise and signal samples, respectively.

Degree of correlation of  $\underline{M}$  sample pairs and  $\underline{P(r \geqq T_2)}$ . It is now assumed that each sample of the photomultiplier signal is independent of any other sample. With this assumption, the distribution of successive correlations of M sample pairs is described by a binomial distribution. The probability of i agreements in M sample pairs representing noise samples is

$$P_{OO}(i,M) = \frac{M!}{(M-i)!i!} P_{OO}^{i} (1 - P_{OO})^{(M-i)}$$
 (9)

Similarly the probability of  $\ j$  agreements out of M sample pairs representing signal plus noise is

$$P_{1,1}(j,M) = \frac{M!}{(M-j)!j!} P_{1,1}(1-P_{1,1})^{(M-j)}$$
 (10)

The sum of noise pair agreements and sample pair agreements is defined as

$$r = i + j \tag{11}$$

The probability that r will be greater than or equal to some preselected number of total agreements may be determined by

$$P(r \ge T_2) = 1 - P(r < T_2)$$

$$= 1 - \sum_{i+j=0}^{T_2-1} \left[ \sum_{i=0}^{M} p_{oo}(i,M) \sum_{j=0}^{M} p_{ll}(j,M) \right]_{(1,2)}$$

where  $T_2$  is the acceptable level of agreements.

Once  $P(r \ge T_2)$  has been determined for a range of values of star magnitude, a range of code lengths and for assumed noise levels, this detection parameter may be combined with the probability of existence of star magnitudes in a single scan (a  $360^{\circ}$  scan with the optical FOV). This combined or joint probability function is defined here as the system probability of response  $P_R(m)$  to all stars in a single scan. A system figure of merit used in this study is

$$R = \frac{\int_{m_1}^{-\infty} P_R(m) dm}{\int_{m_1}^{-\infty} P_R(m) dm}$$
 (13)

which is a measure of the system response to stars brighter than  $\[mu]$  compared to system response to all star magnitudes. Ideally this parameter would be equal to unity.

This discussion is based upon several assumptions which are listed here in summary:

- (a) The azimuth and elevation code groups are independent but background noise is a function of the open slits in both code groups.
- (b) The sample intervals of each code group are independent.
- (c) Distribution of electrons produced at the cathode of the photomultiplier is a Poisson distribution.
- (d) Statistics of signal and noise from all sources are additive.
- (e) Electronic noise such as Johnson noise is negligible compared to background noise, dark current noise, and signal dependent noise.
- (f) Probability of more than one star of magnitude brighter than m<sub>1</sub> (threshold magnitude) occurring in the telescope field of view is negligible.

Advantages and disadvantages of coding. The advantages of coding lie principally with the enhancement of detection capability through multiple signal pulses for a given star image. The described technique will provide this advantage and yet retain the precision of angular resolution of a single pulse indication of a star image.

However, as code length is increased, system noise will increase proportionally since the

effective background noise as sensed by the photomultiplier is proportional to the effective field of view which in turn is proportional to the number of transparent coded openings in the reticle. The optimum number of code elements for a given system is dependent upon the noises of the system and the desired minimum detectable star magnitude.

A second disadvantage in coding results from using very short code lengths. This disadvantage is the inability to design an unambiguous code group, that is to say, a code group which will result in a cross correlation function with a uniquely defined peak. The question of desirability of coding in a given system must be answered through an analysis which includes the particular system parameters and environment.

#### PERFORMANCE ESTIMATES OF A SPECIFIC SYSTEM

A sample design is presented here to demonstrate the effects on system performance due to increasing code lengths. Certain assumed noise conditions and fixed system parameters are listed below:

### Assumed System Parameters and Noise Conditions

- (a) Telescope aperture 40 cm<sup>2</sup>
- (b) Telescope field of view  $6^{\circ} \times 6^{\circ}$
- (c) Field of view of a basic transparent slit of the reticle  $6^{\circ}$  × 0.025°
- (d) Photomultiplier cathode luminous sensitivity 25  $\mu a/lumen$
- (e) Photomultiplier dark current (equivalent noise input)  $5 \times 10^{-13}$  lumens  $\sec^{1/2}$
- (f) Spin rate of vehicle 90°/sec
- (g) Width of transparent slit 0.025°
- (h) Star density is assumed to be that which would result from a  $6^{\circ}$  ×  $360^{\circ}$  scan of the mean galactic latitude
- (i) Background light equivalent of 160, 500, and 1,000 tenth magnitude stars per square degree

In the construction of the signal and noise models the expected values of signal and noise photoelectrons were computed as a function of the assumed system parameters, by use of equations (1) through (4). As previously stated the distribution of these electrons is considered to be a Poisson distribution. Visual magnitudes ranging from zero to sixth were assumed for the star signals. Code lengths were varied from two to twenty-four elements.

In the calculations the approach taken was to set  $T_2$  at 0.75n for all code lengths except for n=2. For this value of code length  $T_2$  was set at n. For ease of calculations the value of  $T_1$  used for each noise case was selected as that value which would result in a probability of 0.90 of 50 percent or greater of all signal plus noise samples exceeding this threshold value with the signal assumed to be a third magnitude star. In the use

of n = 2, this percentage was set at 100 percent for obvious reasons. With this consideration values of  $P_{11}$  were determined by use of equation (10) for various code lengths using the three assumed background noise conditions and a signal representing a third magnitude star. Using these values of  $P_{11}$  the required values of  $T_1$  were determined by use of equation (7).

Using these first threshold values  $P_{11}$  may be determined for any signal plus noise combination and likewise  $P_{00}$  may be determined for any noise-only condition. After  $P_{11}$  and  $P_{00}$  have been determined for a set of signals and an assumed noise case  $P(r \ge T_2)$  may be determined by equation (12) which is repeated here,

$$P(r \ge T_2) = 1 - \sum_{i+j=0}^{T_2-1} \left[ \sum_{i=0}^{M} p_{oo}(i,M) \sum_{j=0}^{M} p_{11}(j,M) \right]$$
 (12)

A representative plot of  $P(r \ge T_2)$  versus signal level (star visual magnitude) is shown in figure 9 for various code lengths and an assumed background noise of 160 tenth magnitude stars per square degree.

## System Probability of Response $P_{R}(m)$

System probability of response  $\,P_R(m)\,$  may be determined by a knowledge of  $\,P(r\geqq T_2)\,$  for various star magnitudes and a knowledge of the probability of occurrence of a star of a given magnitude  $\,P(m)\,$ . For a  $6^{\circ}\times 360^{\circ}\,$  scan of the mean galactic latitude, the probability of occurrence  $\,P(m)\,$  of a star of a given magnitude may be determined from figure 6, where m may assume values such as 0, 1, 2, 3, . . . m representing various visual magnitude stars. The probability of system response to a star of a given magnitude may then be determined by the relationship

$$P_{R}(m) = P(m)P_{m}(r \ge T_{2})$$
 (14)

Figures 10, 11, and 12 represent  $P_R(m)$  versus m as a function of code length for the assumed background noise conditions of 160, 500, and 1,000 tenth magnitude stars per square degree, respectively. A qualitative measure of system performance may be determined by examination of the relationship between system response to stars of magnitude brighter than third magnitude and system response to stars of all magnitude. This relationship can be expressed in ratio form as

$$R = \frac{\int_{3}^{-\infty} P_{R}(m) dm}{\int_{\infty}^{-\infty} P_{R}(m) dm}$$
 (13)

This ratio is shown plotted as a function of code length for the three assumed background noise conditions in figure 13. As seen in this figure the systems shows continuing improvement for increasing code length for the considered noise conditions and for the values of n considered. However, the rate of improvement is seen to decrease for code lengths in excess of four elements. The increasing improvement in performance indicated by this figure is somewhat misleading for n very large, since total system noise is a function of the number of code elements. The effect of continuously increasing would be a dropping off of the curves of figure  $1^{l_{\downarrow}}$ . For the particular assumed problem a best choice of code length would be of the order of eight elements. This choice is based on the diminishing improvement in performance and increasing complexity of design as the code length is increased above this value. Increased improvement in performance could be achieved by the use of longer codes for systems which are dark current noise limited rather than background noise limited as for the design case presented here.

#### CONCLUDING REMARKS

A method of measuring the attitude angles of a spin stabilized spacecraft has been presented. This method basically consists of a map matching procedure in which a strip star map generated about the equator of the vehicle is correlated with a known reference star map of the celestial sphere. This correlation process results in the determination of the orientation of the vehicle spin axis and roll angle about this axis with respect to the celestial sphere.

The star mapper and necessary data processing mechanization has been described in block diagram form. A general discussion of the system sensitivity and factors which limit system performance has been presented. The unique feature of this system lies in the use of coded optical slits which produces a coded pulse group for each star scanned by the star mapper. This feature provides enhancement to detection capabilities, reduces the probability of false responses due to noise, and yet retains the degree of resolution obtainable through the use of a single slit.

Performance estimates for a typical design case have been determined for a range of signal and noise conditions. Effects on these performance estimates are illustrated for various code lengths and it has been shown that selection of an optimum code length is a function of exact knowledge of the noise of the system. In this discussion a choice of code length has been made based upon the observable diminishing improvement and complexity of mechanization as a function of code length.

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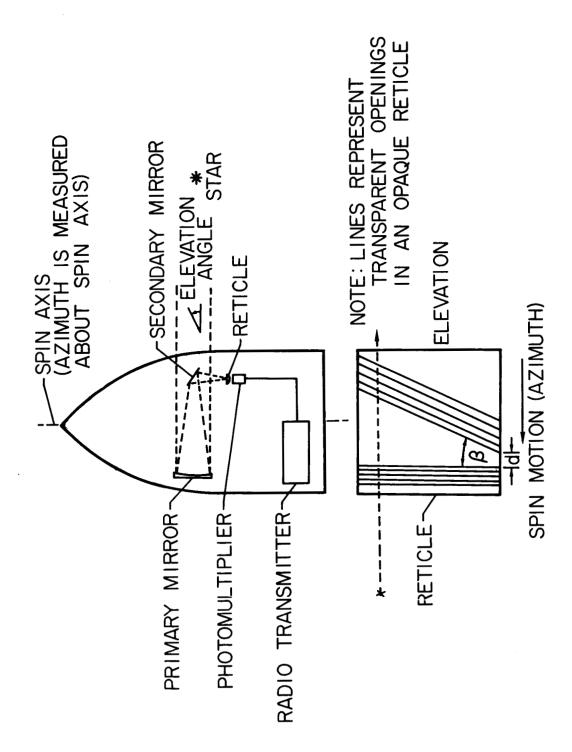


Figure 1.- Simplified drawing of star mapper.

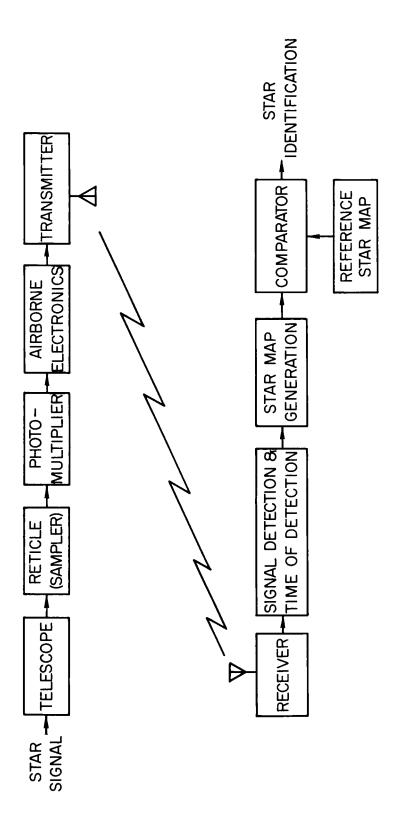


Figure 2.- Simplified system block diagram.

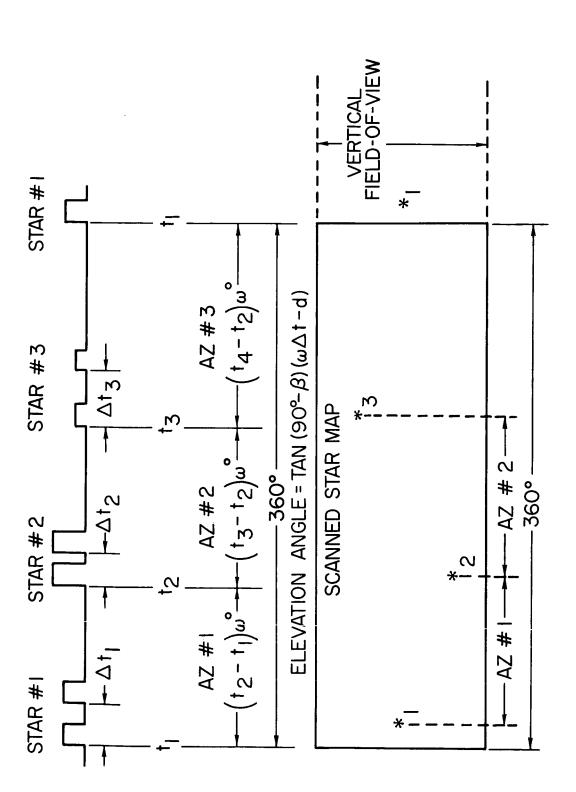


Figure 3.- Computed star map.

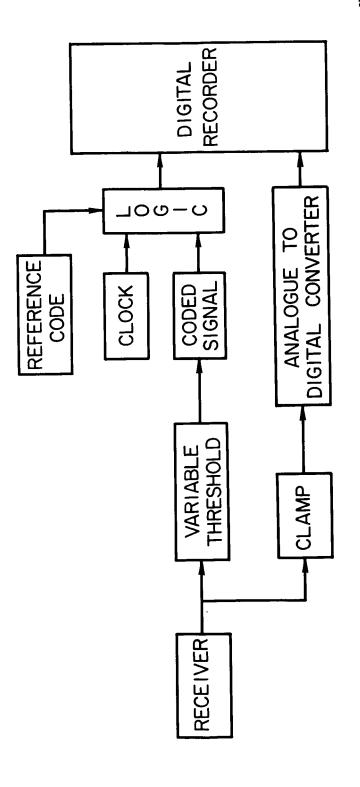


Figure 4.- Data processing block diagram.

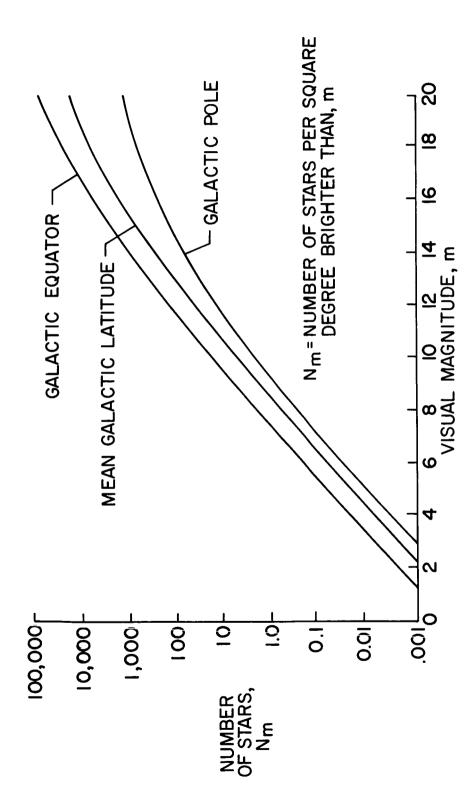


Figure 5.- Spatial density of stars.

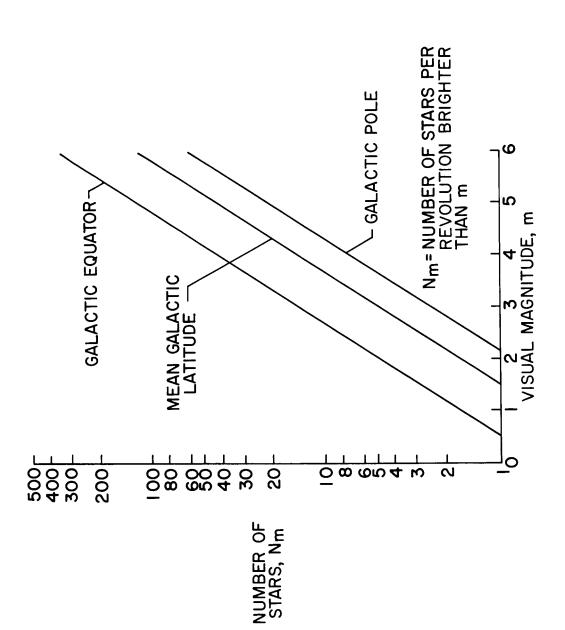


Figure 6.- Spatial density of stars per revolution. (Based on 2,160 square degrees.)

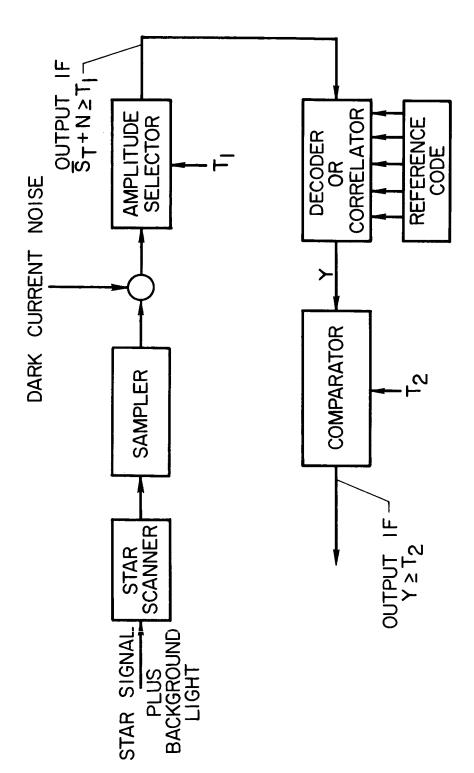


Figure 7.- Block diagram of detection process.

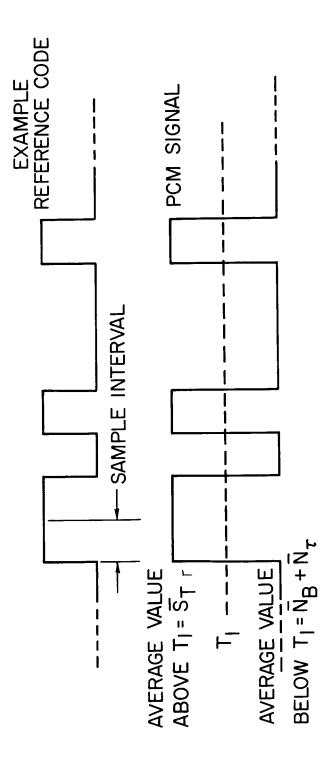


Figure 8(a).- PCM signal and reference code.

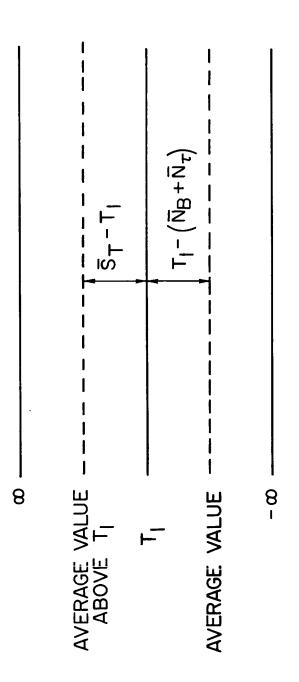


Figure 8(b).- Level limits and average values of sample intervals.



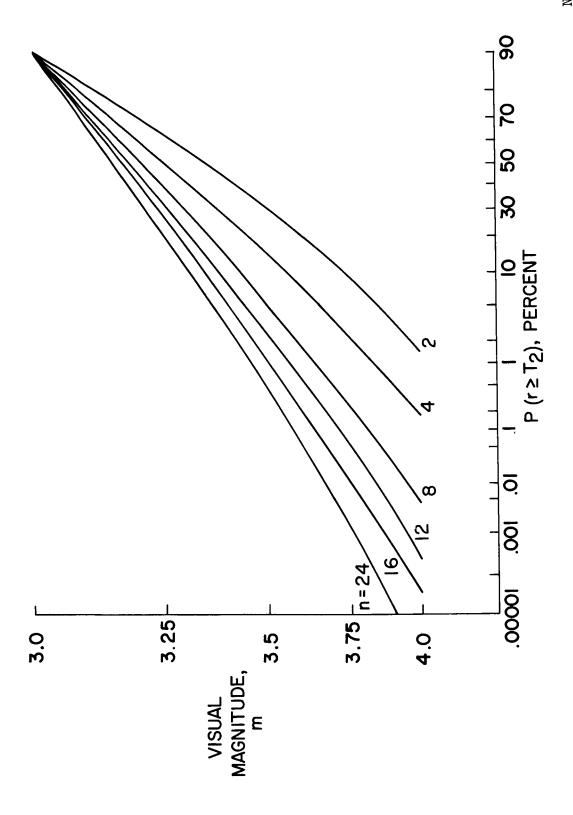
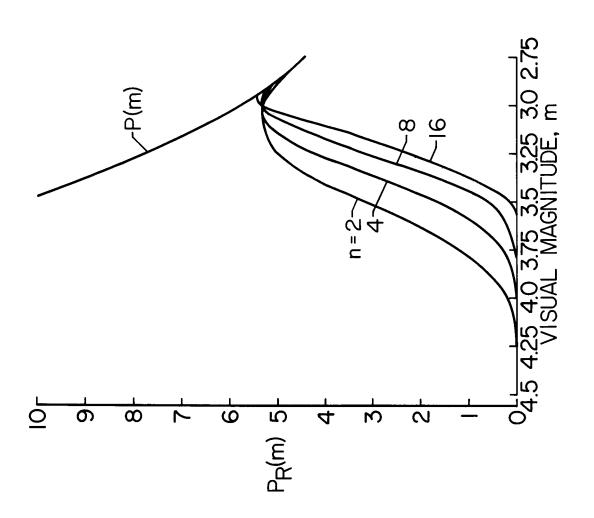


Figure 9.-  $P(r \ge T_2)$  versus visual magnitude. (Background of 160 tenth magnitude stars per square degree.)



(Background of 160 tenth magnitude stars per square degree.) Figure 10.-  $P_{R}(m)$  versus m.

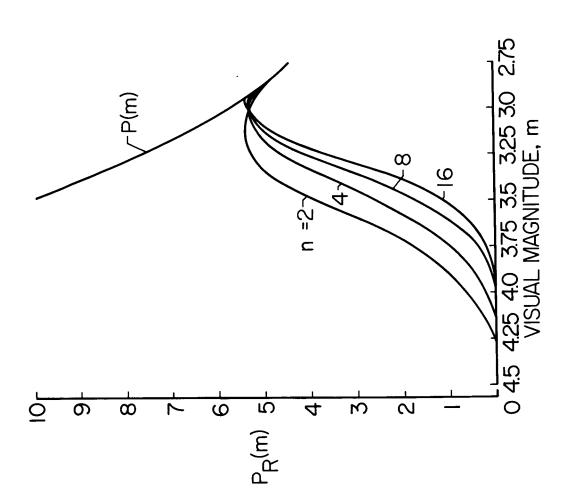


Figure 11.-  $P_R(m)$  versus m. (Background of 500 tenth magnitude stars per square degree.)

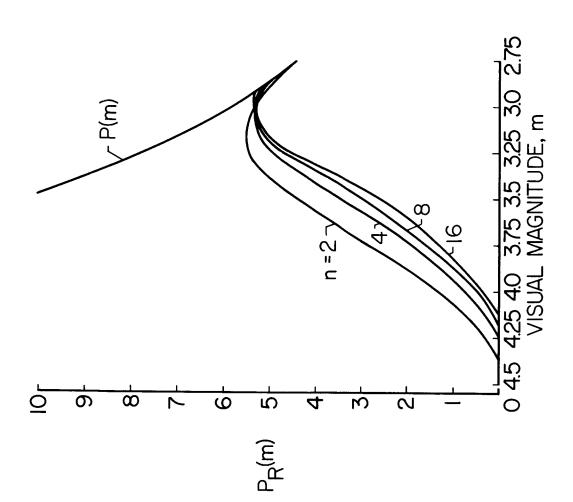


Figure 12.-  $P_R(m)$  versus m. (Background of 1,000 tenth magnitude stars per square degree.)

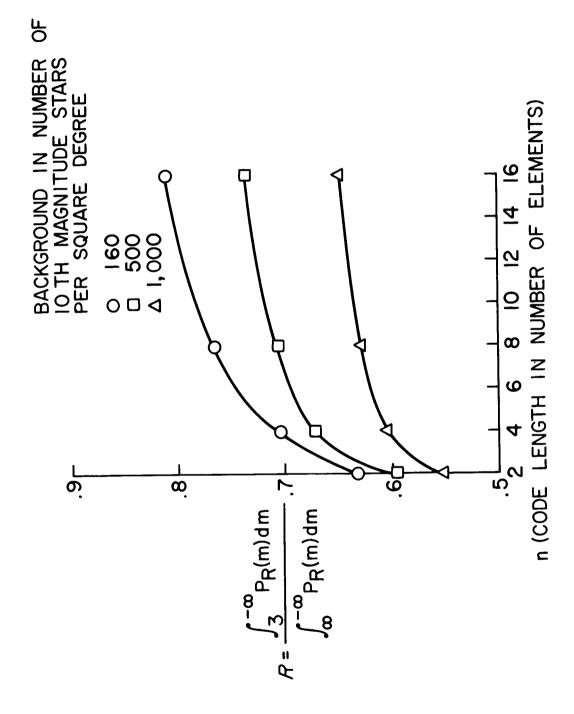


Figure 13.- System performance versus code length.